Northwestern University Department of Physics and Astronomy

PHYSICS 428-2 QUANTUM FIELD THEORY II

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Course Webpage: `http://www.hep.anl.gov/ian/teaching/QFTII/QFT_Winter09.html`

ASSIGNMENT #1

Due at 3 PM, January 12th

(One page and five problems in total.)

Reading Assignments:
Sections 6.1 and 6.2 of Peskin and Schroeder.

Problem 1
Do Problem 6.1 in Peskin and Schroeder.

Problem 2
Use L to denote the number of loops in a Feynman diagram, I the number of internal lines, V the number of vertices, and E the number of external lines.
(a) Restore the Planck constant $\hbar$ in the quantum field theory and show that, for a fixed E, the power of $\hbar$ associated with a Feynman diagram is $L-1$.
(b) Consider a scalar ϕ^4 theory where $\mathcal{L}_{int} = \lambda\phi^4$ and show that, for a fixed E, the power of λ associated with a Feynman diagram is $L-1+E/2$. (E must be even, why?)

Problem 3
Here are two ways to prove the general Feynman's parameters without using the induction.
(a) Prove Eq. (6.41) using the identity

$$\frac{i}{(A+i\epsilon)} = \int_0^\infty d\alpha\, e^{i\alpha(A+i\epsilon)}$$

(b) Consider the following definition of the Gamma function

$$\Gamma(x) = \int_0^\infty dt\, t^{x-1}e^{-t}, \quad \Gamma(n+1) = n!.$$

Derive a variation of the above by changing the variable $t \to At$ and use it to prove Eq. (6.42).

Problem 4
Prove the Chisholm identity

$$\gamma^\mu\gamma^\nu\gamma^\rho = g^{\mu\nu}\gamma^\rho - g^{\mu\rho}\gamma^\nu + g^{\nu\rho}\gamma^\mu + i\epsilon^{\mu\nu\rho\sigma}\gamma_\sigma\gamma^5.$$

Problem 5
Prove the surface area of an n-dimensional unit sphere is

$$\int d\Omega_n = \frac{2\pi^{n/2}}{\Gamma(n/2)}.$$

This expression allows us to analytic-continue into arbitrary dimensionality d in chapter 7.